Continuous Uniform Distribution

**User Input:**

* The user is prompted to enter the lower limit (**a**) and higher limit (**b**) for the continuous uniform distribution.

Probability Density Function (PDF):

* The **continuous\_uniform\_pdf** function defines the Probability Density Function (PDF) for the continuous uniform distribution.
* The PDF is defined as 1 / (b - a) for values of x between a and b (inclusive), and 0 elsewhere.

Function Signature:

* + The function continuous\_uniform\_pdf takes three parameters:

x: The variable for which the PDF is evaluated.

a: The lower limit of the uniform distribution.

b: The upper limit of the uniform distribution.

Explanation of the PDF Formula:

* + The PDF of a continuous uniform distribution is a constant value within the interval [a, b] and is zero outside this interval.

Implementation using NumPy's np.where:

* + np.where((a <= x) & (x <= b), 1 / (b - a), 0) utilizes NumPy's np.where function to apply the PDF formula conditionally.
  + (a <= x) & (x <= b) checks if x falls within the interval [a, b].
  + If the condition is true, it evaluates to 1 / (b - a), representing the constant PDF within the interval.
  + If the condition is false, it evaluates to 0, representing the PDF outside the interval.

Return Value:

* + The function returns an array of PDF values corresponding to the input values of x.
  + The values are 1 / (b - a) within the interval [a, b] and 0 outside this interval.

Cumulative Distribution Function (CDF):

* The **continuous\_uniform\_cdf** function defines the Cumulative Distribution Function (CDF) for the continuous uniform distribution.
* The CDF is defined as 0 for values of x less than a, (x - a) / (b - a) for values between a and b (inclusive), and 1 elsewhere.

Function Signature:

The function continuous\_uniform\_cdf takes three parameters:

x: The variable for which the CDF is evaluated.

a: The lower limit of the uniform distribution.

b: The upper limit of the uniform distribution.

Explanation of the CDF Formula:

* The CDF of a continuous uniform distribution increases linearly within the interval [a, b] and remains constant at 1 outside this interval.

Implementation using NumPy's np.where:

* np.where(x < a, 0, np.where(x <= b, (x - a) / (b - a), 1)) utilizes NumPy's np.where function to apply the CDF formula conditionally.
* x < a checks if x is less than the lower limit a. If true, it evaluates to 0.
* x <= b checks if x is less than or equal to the upper limit b. If true, it evaluates to (x-a)​/(b-a)
* If both conditions are false, it evaluates to 1, representing the constant value of the CDF outside the interval.

Return Value:

* The function returns an array of CDF values corresponding to the input values of x.
* The values are 0 for x < a, increase linearly from 0 to 1 with the interval [a, b], and become 1 for x > b.

Variance and Expectation (Mean):

* The **continuous\_uniform\_variance** function calculates the variance of the continuous uniform distribution.
* The **continuous\_uniform\_expectation** function calculates the expectation (mean) of the continuous uniform distribution.

Function Signature:

* The function continuous\_uniform\_variance takes two parameters:

a: The lower limit of the uniform distribution.

b: The upper limit of the uniform distribution.

Explanation of the Variance Formula:

* The variance of a continuous uniform distribution is calculated using the formula (b-a)\*\*2 / 12.

Implementation:

* The implementation directly applies the formula using Python arithmetic.
* ((b - a)\*\*2) / 12 calculates the variance based on the given lower and upper limits.

Return Value:

* The function returns the calculated variance of the continuous uniform distribution.

Plotting Functions:

* Two plotting functions are defined to visualize the PDF and CDF of the continuous uniform distribution.
* The **plot\_continuous\_uniform\_pdf** function plots the PDF, and the **plot\_continuous\_uniform\_cdf** function plots the CDF.

**plot\_continuous\_uniform\_pdf**

Function Signature:

* The function plot\_continuous\_uniform\_pdf takes two parameters:

a: The lower limit of the uniform distribution.

b: The upper limit of the uniform distribution.

Generate x-values for Plotting:

* x\_values = np.linspace(a-2, b+2, 1000) generates a range of 1000 equally spaced x-values for plotting. The range is extended by 2 units on both sides of the interval [a, b] to ensure a clear visualization.

Plotting PDF:

* plt.plot(x\_values, continuous\_uniform\_pdf(x\_values, a, b), label='PDF') plots the Probability Density Function (PDF) using the continuous\_uniform\_pdf function.

Plot Customization:

* plt.title, plt.xlabel, plt.ylabel, and plt.legend are used for customizing the plot with appropriate labels and a legend.

Show the Plot:

* plt.show() displays the generated plot.

**plot\_continuous\_uniform\_cdf**

Function Signature:

* The function plot\_continuous\_uniform\_cdf takes two parameters:

a: The lower limit of the uniform distribution.

b: The upper limit of the uniform distribution.

Generate x-values for Plotting:

* x\_values = np.linspace(a-2, b+2, 1000) generates a range of 1000 equally spaced x-values for plotting. The range is extended by 2 units on both sides of the interval [a, b].

Plotting CDF:

* plt.plot(x\_values, continuous\_uniform\_cdf(x\_values, a, b), label='CDF', color='green') plots the Cumulative Distribution Function (CDF) using the continuous\_uniform\_cdf function. The CDF is plotted in green.

Plot Customization:

* plt.title, plt.xlabel, plt.ylabel, and plt.legend are used for customizing the plot with appropriate labels and a legend.

Show the Plot:

* plt.show() displays the generated plot.